**COMP 7270 Assignment 4 200 points No late submissions!**

**Due by 11:59 PM Tuesday 05/02**

**Upload your submission well before this deadline. Even if you are a few minutes late, as a result of which Canvas marks your submission late,** **your assignment may not be accepted**.

Instructions:

1. This is an individual assignment. You should do your own work. **Any evidence of copying will result in a zero grade and additional penalties/actions.**
2. Late submissions **will not** be accepted unless prior permission has been granted or there is a valid and verifiable excuse.
3. **Think carefully; formulate your answers, and then write them out concisely** using English, logic, mathematics and pseudocode (no programming language syntax).
4. Algorithms should be provided in numbered pseudocode steps.
5. **Type your answers in this Word document and submit it. If that is not possible, use a word processor to type your answers as much as possible (you may hand-write/draw equations and figures), turn it into a PDF document and upload**.

**1. (5.4-2) in CLRS book.**

Suppose that we toss balls into b bins until some bin contains two balls. Each toss is independent, and each ball is equally likely to end up in any bin. What is the expected number of ball tosses?

**Answer:**

**We suppose the two balls into the same bin is “q” and “ p ”, m and n is independent. Suppose** **Yqp = H{ball q and ball p in same bin}. If we repeat the process of tossing n times ,**

**the total numbers of bins are b, so the expected value E[Yqp] is (1/b).**

**since there are two balls q and p, and the number of bins are b, so the chance is**

**(1/b \* 1)=(1/b)**

**use this expectation in the equation**

**E[Y]=E[]**

**=**

**=**

**= ⇒ so n(n-1) = 2b ⇒**

**since the number should be no less than 1.**

**The expect number of balls tossed into b bins until some balls is**

**2. (5.4-6) in CLRS book.**

**Answer:**

**Let Xi be the indicator variable that bin i is the only one bin which is empty and X be the random variable that gives the number of empty bins, so**

**= n()n  =**

**let Xi be the indicator variable that bin i contains exactly 1 ball after all balls are tossed and X be the random variable that gives the number of bins containing exactly 1 ball, so :**

**so the expected number of bins with exactly one ball is**

**3. (5-1) in CLRS book.**

**Answer : (1)**

Assuming that represents the value of the given counter after the increment.

Assuming that represents the increment value of the given counter after the increment.

Then we have

Then

Since the increased value is if it’s increased and it’s 0 , it’s not increased.

We also have the probability of the incrementing the counter, which is and the probability of not incrementing the counter.

So,

Thus,

so, after INCREMENT operations ,the expected value represented by the counter is exactly .

b.

Solution:

Assuming that denotes the variance, then we have to calculate , where Y defined in part a.

Then:

Since

Then

Thus, the variance in the value shown by the register after increment operations is .

**4. (Iterated Monte Carlo algorithm)**

Say we have a Monte-Carlo randomized algorithm A which may return the wrong answer, but the Pr[A is wrong]≤  ,where n is the size of the input and  is a constant. How many repetitions of algorithm A are required to reduce the error probability to ?

Answer:

Since algorithm A is a Monte-Carlo algorithm, and the probability of A is wrong is .

The probability of A is right is .

Assume that there are repetition of algorithm A are required to reduce the error probability to .

There are repetitions of algorithm A are required to reduce the error probability to .